

# Common Limits Cheat Sheet

Complete common limits cheat sheet with standard limits, indeterminate forms, L'Hôpital's rule, and limits at infinity. Free PDF download for calculus students.

## Important Standard Limits

### Sine Limit

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$

*Fundamental trig limit*

### Cosine Limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x} = 0$$

### Cosine Limit 2

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2} = \frac{1}{2}$$

### Euler's Number

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

### Euler's Number 2

$$\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$$

### Natural Log

$$\lim_{x \rightarrow 0} \frac{\ln(1 + x)}{x} = 1$$

### Exponential

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

## Limits at Infinity

### Polynomial Growth

$$\lim_{x \rightarrow \infty} \frac{1}{x^n} = 0 \quad (n > 0)$$

### Exponential Dominates

$$\lim_{x \rightarrow \infty} \frac{x^n}{e^x} = 0$$

$e^x$  grows faster than any polynomial

### Polynomial Dominates Log

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{x^n} = 0 \quad (n > 0)$$

### Ratio of Polynomials

$$\lim_{x \rightarrow \infty} \frac{a_n x^n + \dots}{b_m x^m + \dots} = \begin{cases} \frac{a_n}{b_m} & n = m \\ 0 & n < m \\ \pm\infty & n > m \end{cases}$$

Compare leading terms

## Indeterminate Forms

### Zero over Zero

$$\frac{0}{0}$$

Try factoring, rationalization, or L'Hôpital's rule

### Infinity over Infinity

$$\frac{\infty}{\infty}$$

Use L'Hôpital's rule

### Zero times Infinity

$$0 \cdot \infty$$

Rewrite as fraction, then use L'Hôpital

### Infinity minus Infinity

$$\infty - \infty$$

Combine fractions or factor

### Zero to Zero

$$0^0$$

Take  $\ln$ , use L'Hôpital, then exponentiate

### One to Infinity

$$1^\infty$$

Use  $\ln$  or write as  $e^{\{\dots\}}$

### Infinity to Zero

$$\infty^0$$

Take  $\ln$ , use L'Hôpital

## L'Hôpital's Rule

### L'Hôpital's Rule

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

Only valid for  $0/0$  or  $\infty/\infty$  forms

## One-Sided Limits

### Right Limit

$$\lim_{x \rightarrow a^+} f(x)$$

*Approaching from the right*

### Left Limit

$$\lim_{x \rightarrow a^-} f(x)$$

*Approaching from the left*

### Limit Exists

$$\lim_{x \rightarrow a} f(x) \text{ exists iff } \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)$$